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Hydraulic Modeling of Trapezoidal High-Velocity Channels

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PURPOSE: This Technical Note presents a recently developed two-dimensional (2D) numerical flow model for trapezoidal high-velocity channels. The model is an adaptation of the existing HIVEL2D code that includes boundary capabilities. Because the sidewalls are sloped, not only the depth, but also the waterline location is unknown. This waterline location is essential in the design of flood control channels and is difficult to determine in supercritical flow wherein the boundary may have significant variation.

INTRODUCTION: High-velocity channels are typically lined flood control channels designed to discharge supercritical flow through specific reaches. The designer of these channels is primarily concerned with the depth of flow for the design discharge. The depth must be known before sidewall heights and minimum bridge soffit elevations are set. Depth determination is complicated by side inflows and boundary features such as contractions, expansions, curves, and obstructions to the flow such as bridge piers, and vehicle access ramps. These boundary features in a supercritical channel cause flow disturbances that can result in a significant increase in the local flow depth.

Local depths are unknown quantities in the solution of the 2D open channel flow equations. If the channel sidewalls are vertical, the plan view domain is defined such that one can easily discretize the domain. However, if the channel has sloping sidewalls, the plan view of the flow domain, as delineated by the water surface/bank interface (i.e. waterline), is not known a priori. The width of the flow field is unknown because it depends on the water surface elevation. Steady state solutions are obtained by time stepping the transient 2D equations. As the computed flow field evolves from the specified initial flow conditions and initial sidewall boundary location to the steady state, the side boundaries of the flow field are adjusted. This constitutes a moving boundary problem.

MODELING CONSIDERATIONS: Determination of the waterline location is a common problem in hydraulic engineering. Waterline movements are present in natural channels and in the tidal flats of estuaries. Solution methods in numerical flow models are either of the fixed grid or moving grid type. Static grid methods are attractive because of their ease of implementation. However, these methods lack precise definition of moving boundaries. A fixed vertical wall is commonly used to represent the waterline of subcritical flow in wide rivers and channels, but does not accurately represent reflection of shocks in supercritical flow. Roig (1994) demonstrates the usefulness of a technique referred to as marsh porosity. The marsh porosity method is included in the RMA2 code (Donnell 1997) as part of the Surface-water Modeling System (SMS). Another popular fixed grid technique employs the volume-of-fluids (VOF) scheme (Hirt and Nichols 1981) where a fluid fraction variable, f, is treated as a transported variable ranging in value from 0 to 1. Boundary cells are identified as those with fractional values of f. There are continuity issues that have not been totally resolved with the VOF.

Another technique, wetting and drying, can be considered a quasi-dynamic grid method. Although the nodal locations remain fixed, the elements included in the flow computations can vary in time as a function of water-surface elevation. Kawahara and Umetsu (1986) demonstrate wetting and drying applicability to river flows. Wetting and drying is an available feature in RMA2 and has been successfully applied to subcritical flow. There are mass and momentum conservation issues that must be addressed when using this method, but more importantly in supercritical flow, the jagged boundary itself will create shocks and steady state convergence may not be obtainable without using a very fine grid near the boundaries.

The Arbitrary Lagrangian-Eulerian (ALE) description of the governing equations is a popular moving grid method of addressing free-surface moving boundary calculations. The ALE technique's name is derived from the fact that nodal displacement can be treated in a Lagrangian manner so that they move with the fluid, they can be held fixed for an Eulerian description, or they can move in a prescribed direction and rate. ALE's flexibility allows customization to the physics of the problem at hand. ALE forms of the depth-averaged equations can be represented using moving finite elements. Akanbi and Katopodes (1988) use the ALE method to model the rapid waterline advancement of flood-wave propagation over initially dry land. ALE is a logical method to model high-velocity channels, which may contain hydraulic jumps and standing waves. The grid moves as the flow domain evolves to steady state and as such, accurately reproduces the smooth waterline even in regions of rapidly varying flow.

HIVEL2D is a finite element model that was initially developed for applications to rectangular high-velocity channels (Berger and Stockstill 1995). The model, which is a component of the SMS, is designed to simulate subcritical and supercritical flow and any transitions between these regimes. The HIVEL2D framework relaxes time-step size limitations and provides numerical stability for supercritical flows. Incorporation of a moving finite element scheme using the ALE method into HIVEL2D provides the numerical stability needed in modeling supercritical flow with moving boundaries (Stockstill, Berger, and Nece 1997).

MODEL EVALUATION: The modified HIVEL2D model, containing the ALE moving grid method, was evaluated by conducting tests to determine the model's ability to simulate real flows. A series of flume tests were conducted at the U.S. Army Engineer Research and Development Center, Coastal and Hydraulics Laboratory. Model parameters were estimated prior to simulation runs and no adjustments to these parameters were made to improve laboratory/simulation comparisons. Model results are compared with two of these flume test data: flow through a bridge crossing and a rectangular-to-trapezoidal transition.

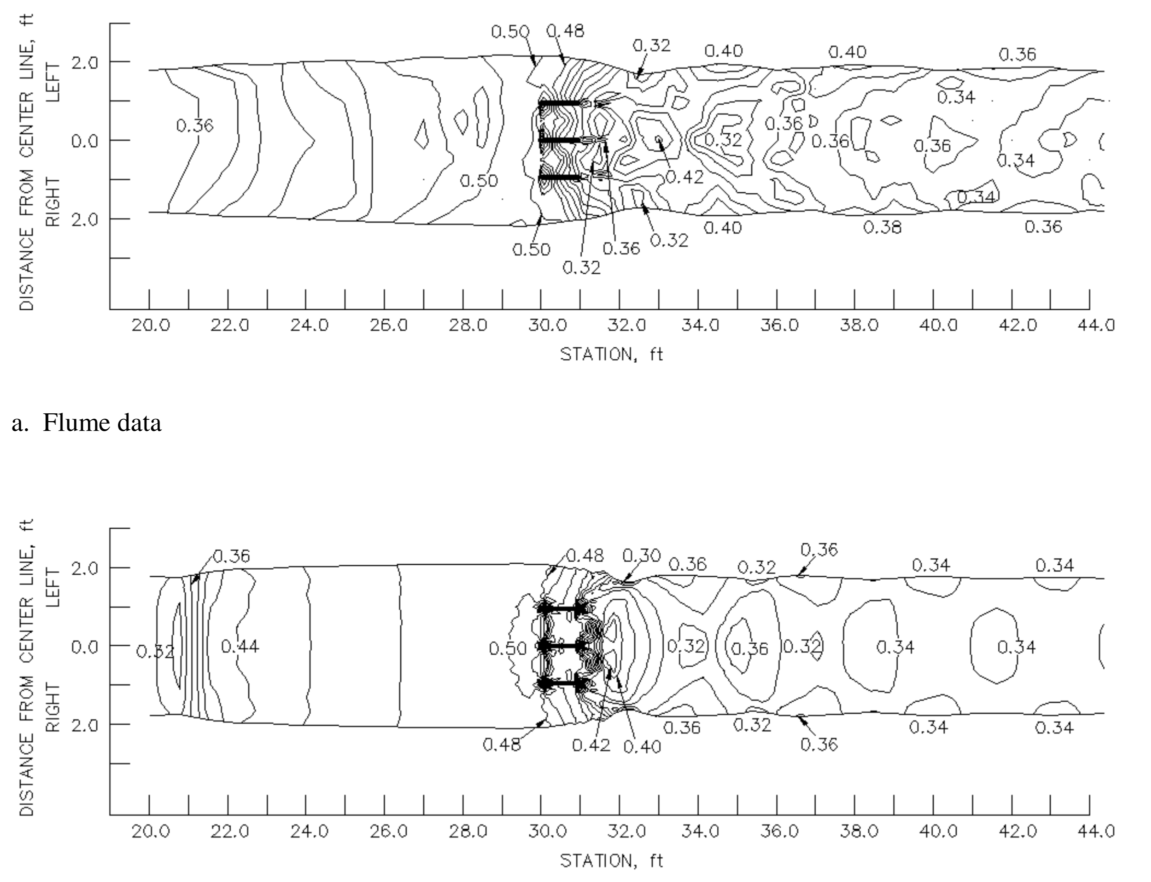
Flow at a Class B Bridge

This experiment was conducted to evaluate the model's ability to simulate the conditions resulting from supercritical flow through bridge piers. Three flow obstructions were placed in a tilting flume having side slopes of 1 vertical on 2.25 horizontal. Each strut had a semicircular nose and tail. The photograph in Figure 1 shows the flow conditions produced by the three piers. The piers cause a choked flow condition resulting in subcritical flow approaching the piers. The

Figure 1. Flow conditions in trapezoidal channel with flow obstructions (looking downstream)

flow accelerates between the obstructions then passes through critical depth resulting in supercritical flow downstream of the piers. Figure 1 shows the multiple oblique standing waves that are generated as the supercritical flow passes around the pier tails. The effects of these standing waves on the shape of the side slope waterlines are also shown on the photograph.

Comparisons are made between the computed and observed flow depths by examination of the depth contours presented in Figure 2. The depth contours illustrate the piers choked the flow so that subcritical flow occurs upstream of the bridge. This phenomenon is commonly referred to as Class B bridge flow in which subcritical flow approaches a bridge over a hydraulically steep channel. An undular hydraulic jump formed in the laboratory flume upstream of the obstructions. The depth-averaged model does not include vertical accelerations and is unable to describe undular jumps. The contours illustrate a slight oscillation upstream of the jump. Downstream of the jump, the model reproduced the water-surface profile of the approach flow observed in the flume. The model did a reasonable job of reproducing the standing waves generated at the pier tails, but the flume results show higher elevations of the waterlines. The most significant result from an engineering standpoint is that the numerical model accurately represented the choked flow condition and the depth of the approaching flow.



a. Flume data

b. Model results

Figure 2. Depth contours (feet above channel bottom) for trapezoidal channel with flow obstructions (Distance is in feet. To convert to meters, multiply feet by 0.3048)

Rectangular-to-Trapezoidal Transition

Evaluation of the model's ability to capture oblique standing waves in trapezoidal channels is made by comparison with data obtained in a tilting flume. The flume had a rectangular channel approaching a 1-lateral-on-6-longitudinal transition to a trapezoidal channel having 1-vertical on 2.25-horizontal side slopes.

Uniform conditions for this experiment produces a Froude number of 2.2 in the transition approach. The flow through the transition is shown in Figure 3. The initial flow conditions and initial grid width were based on normal flow conditions. Boundary nodes at the waterline/ transition-wall intersection on each side of the channel were allowed to move in the direction parallel to the vertical transition walls. As the water-surface elevation at this point increased or decreased, the intersection point moved downstream or upstream, respectively. All other side-slope boundary nodes were specified to move in the direction of the maximum side slope.

The final grid is shown in Figure 4 as a water-surface mesh. As the flow expands at the upstream end of the transition, the flow on each side meets the steep side slopes, producing a rise in the water surface within the portion of the transition located over the sloping sidewalls. The flow depth near the channel center line decreases beginning at the upstream end of the transition because of the expansion of the channel. The water run-up on the side slopes and the depression along the center line are followed by a depression along the side slopes and a rapid increase along the center line as the standing waves generated at the side slopes intersect. These oblique standing waves result in varying waterlines along the side slopes.

Computed and observed water-surface elevations are shown in Figure 5. There is a phase difference in the computed and observed wave patterns. The computed location of the first wave peak is slightly downstream of that measured in the flume. The error is probably due to a combination of violations of both the geometrically mild slope and the hydrostatic pressure assumptions. It appears that the validity of the depth-averaged equations varies along trapezoidal channels.

CONCLUSIONS: The HIVEL2D model has been extended to account for moving boundary effects. The numerical model was tested by comparison of simulation results with laboratory data. Simulation results demonstrate the model's ability to solve the domain limit and the flow variables within the domain. The extended HIVEL2D provides a unique capability of modeling trapezoidal-shaped high-velocity channels.

Future efforts will be directed to designing a user-friendly interface for the extended HIVEL2D code. This will simplify the additional boundary condition input that the new HIVEL2D code requires. The graphical user interface will be incorporated within the existing SMS package.

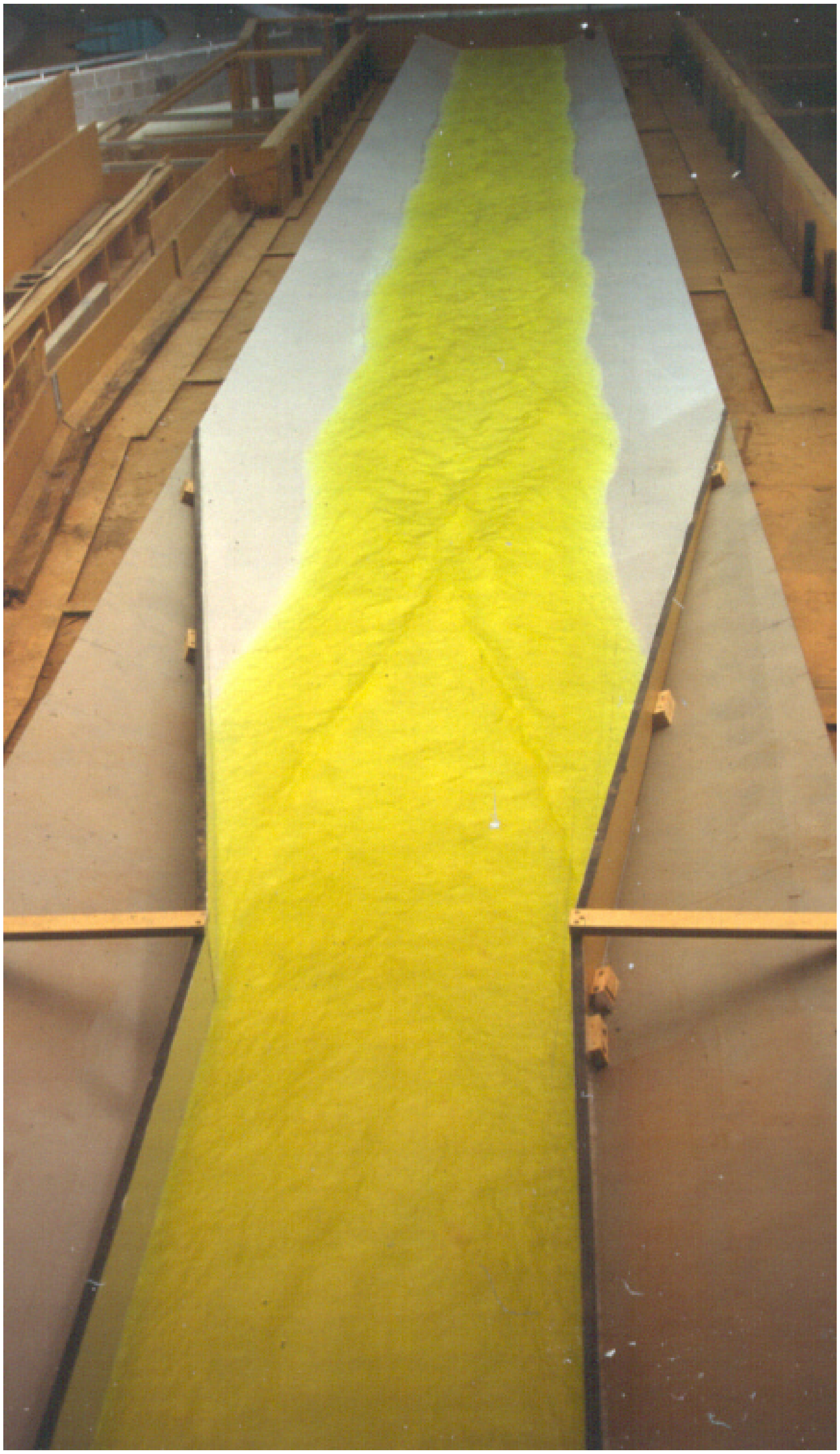


Figure 3. Flow conditions in transition (looking downstream)

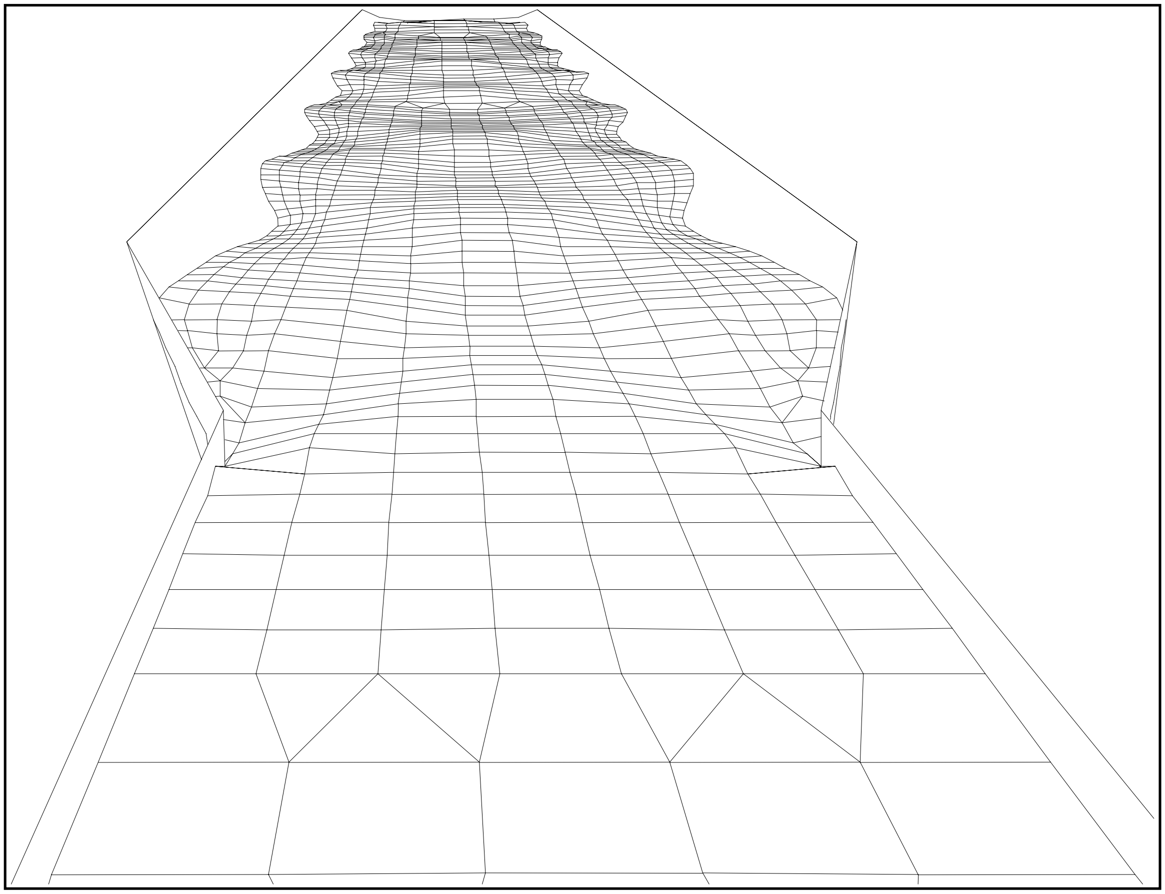
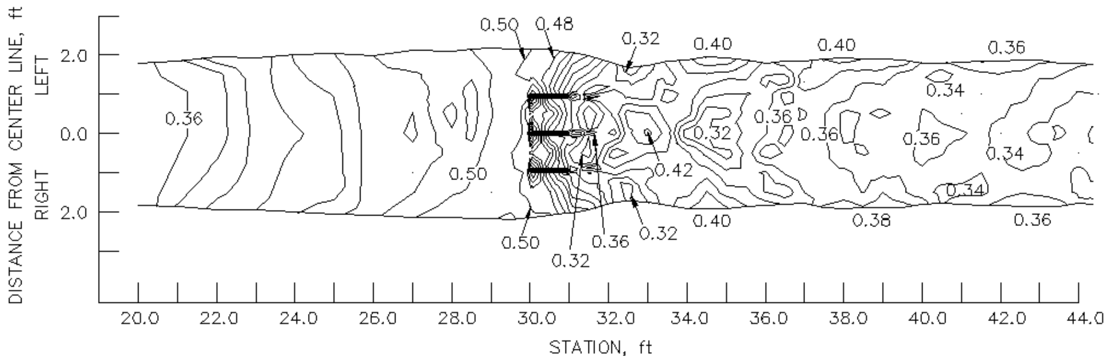
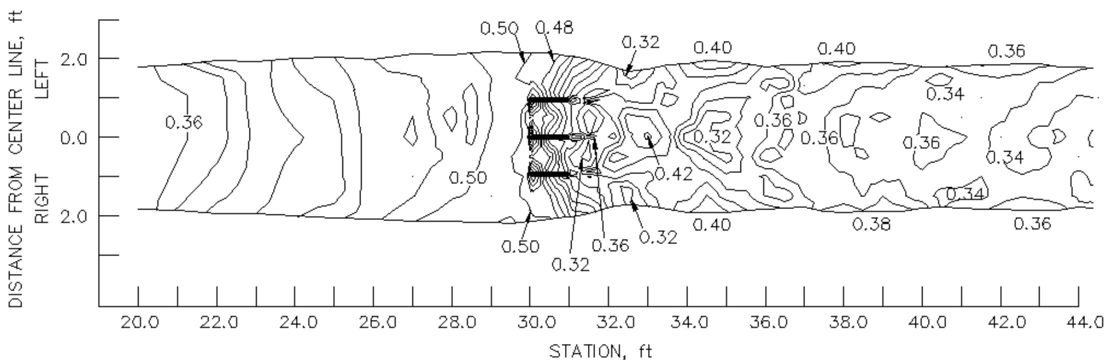


Figure 4. Computed water-surface mesh for transition



a. Flume data



b. Model results

Figure 5. Depth contours (feet above channel bottom) for transition (Distance is in feet. To convert feet to meters, multiply feet by 0.3048)

ADDITIONAL INFORMATION: For additional information contact Dr. Richard L. Stockstill, Navigation Branch, Coastal and Hydraulics Laboratory, U.S. Army Engineer Research and Development Center at 601-634-4251 or e-mail *stocksr@wes.army.mil*.

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